International Journal of Educational Development

E-ISSN: 3046-8159
P-ISSN: 3047-1893

Research Article

Design of the Learning Trajectory of One-Variable Linear Equations in Junior High School Through a Realistic Mathematical Approach to Generative Models

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Abstract: This study aims to identify students' learning obstacles in the process of understanding one-variable linear equations in grade VII, develop an appropriate learning trajectory using a Realistic Mathematical Education (RME) approach, and produce an effective learning design to reduce these obstacles through generative models. The research adopts a design research methodology consisting of three main stages. The first stage, preparation for the experiment, includes a literature review and analysis of teachers' instructional practices to develop a Hypothetical Learning Trajectory (HLT) for one-variable linear equations. The second stage, design experiment, involves testing the designed learning activities through a pilot experiment and teaching experiment. The third stage, retrospective analysis, compares actual classroom observations with the initial HLT to refine and validate the learning trajectory. The research subjects include seven grade VII students in the pilot experiment and 44 students from SMP Negeri 2 Mataram in the teaching experiment. Data collection methods include tests, observations, interviews, and documentation, with retrospective analysis used to generate a validated Local Learning Trajectory (LLT). The findings reveal several epistemological obstacles experienced by students, such as difficulties in applying fundamental arithmetic concepts (addition, subtraction, multiplication, division) within the context of linear equations. Students also struggled with concept recognition, representation, and interpretation of linear equations in various forms. The developed LLT proved effective, as students demonstrated improved understanding and were able to follow the learning sequence meaningfully. Retrospective analysis confirmed that the LLT successfully addressed and reduced students' learning obstacles in mastering one-variable linear equations.

Keywords: Epistemological Obstacles; Hypothetical Learning Trajectory; Junior High School; Learning Obstacles; Trajectory Design.

Received: December 22, 2025
Revised: January 11, 2026
Accepted: January 28, 2026
Published: January 31, 2026
Curr. Ver.: January 31, 2026

1. Introduction

Education plays a strategic role in shaping the development of individuals and society as a whole. One of the main paths of achievement is through the learning process in the classroom, which becomes an educational interaction between teachers and students (Rustaman, 2001). The success of education is greatly influenced by teachers' skills in managing the learning environment so that it is conducive and effective (Sarea et al., 2022). Teachers are not only in charge of delivering material, but also play a role in directing, facilitating, and creating a learning atmosphere that encourages active participation of students. Thus, the learning process requires careful planning starting from the planning, implementation, to evaluation stages (Sarea et al., 2022).

Mathematics is one of the subjects taught at all levels of education because of its essential function in training logical, critical, creative, and analytical thinking skills (Astuti & Wijaya, 2020). The role of mathematics includes the development of reasoning and problem-solving skills, both for academic purposes and practical needs in daily life. However, the abstract nature of mathematics often causes difficult perceptions for students, so a learning approach is needed that is able to bridge abstract concepts into more concrete and meaningful (Thalhah et al., 2019).

One-variable linear equations (PLSV) are one of the basic materials in algebra taught in grade VII of junior high school (Jupri et al., 2014; Khuluq, 2015). Mastery of PLSV becomes an important foundation for understanding advanced algebraic material. Unfortunately, a number of studies show that the understanding of the concept of PLSV among students is still low. Students often make procedural errors, are unable to connect concepts, and misunderstand the story (Sumiyati et al., 2019; Jumiati & Zanthy, 2020). These difficulties not only hinder the achievement of learning objectives, but also affect their readiness to learn the next math concept.

Various studies have identified the source of *learning obstacles* in PLSV, ranging from weak mastery of basic concepts, errors in operating equations, to the inability to create mathematical models of real problems (Singh & Sharma, 2018; Rayhan & Eyus, 2022; Rivera & Lopez, 2019). These barriers can be grouped into three categories, namely ontogenic barriers (students' mental readiness), didactic barriers (inappropriate learning strategies), and epistemological barriers (limited conceptual understanding) (Brousseau, 2002). To overcome this, a learning design is needed that is able to identify these barriers and develop appropriate learning trajectories.

The Realistic Mathematics Approach (RME) is seen as appropriate to teach PLSV because it views mathematics as *human activity* that is built through real experience and context (Freudenthal, 2002). This approach encourages students to construct their own knowledge with the guidance of the teacher, so that learning becomes more meaningful. Furthermore, generative learning models (Lestari et al., 2020; Sadewi et al., 2020; Wena, 2020) supports active learning through four main phases, namely exploration, focusing, challenge, and application. This model is in line with the principle of *Hypothetical Learning Trajectory* (HLT) which directs students to achieve gradual understanding (Simon, 1995; Gravemeijer, 1994).

The conditions at SMP Negeri 2 Mataram show the need for PLSV learning innovation. The average score of students in this material is still relatively low in the last two school years, while teachers have not implemented RME and tend to rely only on package books. This raises suspicions that the learning trajectory used is not appropriate and effective. Therefore, this research is focused on three main objectives: (1) identifying students' *learning obstacles* in learning PLSV, (2) developing *a Local Learning Trajectory* (LLT) based on an appropriate generative model RME, and (3) producing a learning design that is able to significantly reduce learning barriers. This research is expected to provide practical benefits for students to overcome learning difficulties, for teachers as a guide for learning design based on empirical evidence, and for other researchers as a reference for further studies.

2. Literature Review

Mathematics learning design is a structured planning process to address various learning challenges. According to Wina Sanjaya (2017), learning design includes the preparation of material plans, resource planning, and the design of evaluation systems that are geared towards achieving learning objectives. Gravemeijer (2004) emphasized that the design of mathematics learning should be carried out systematically and contextually, while Dick & Carey (2001) emphasized the importance of problem solving and the development of students' abilities by utilizing relevant models. The main principles according to Cobb & Gravemeijer (2008) are real-context-based, problem-oriented, using the right mathematical models, and developing an in-depth understanding of concepts. In the learning process, the concept of learning *trajectory* was introduced by Simon (1995) through the term *Hypothetical Learning Trajectory (HLT)* which includes learning objectives, designed activities, and predictions of the development of student understanding. In the One Variable Linear Equation (PLSV) material, the learning trajectory is structured to help students understand concepts gradually through contextual activities that encourage knowledge construction.

PLSV learning is an important part of algebra material that must be mastered by junior high school students, because it is the basis for a more complex understanding of algebra (Khuluq, 2015). However, many students experience *learning obstacles* such as difficulty representing problems into mathematical models, making generalizations, and applying concepts to various contexts (Brousseau, 2002). These obstacles can come from ontogenic, didactic, or epistemological factors. To overcome this, a contextual and participatory learning approach is needed. One suitable approach is the Realistic Mathematics Approach (PMR), which views mathematics as a human activity and emphasizes the construction of knowledge through problem-solving that is close to students' lives (Laurens et al., 2018). According to Fauzan (2002), the characteristics of PMR are the use of real context, active involvement of students in finding concepts, interactive learning, and the role of teachers as facilitators.

In its application, PMR can be combined with a generative learning model based on constructivism. This model encourages students to build new knowledge by relating it to initial knowledge through the process of organizing, elaborating, and integrating information (Fiorella & Mayer, 2015; Lestari et al., 2020). The syntax of generative learning includes orientation, concentration, concept formation, and application (Utomo, 2020). Its advantages are to create an active learning atmosphere, provide freedom of thought, and improve students' reflective abilities (Sadewi et al., 2020), although it takes a relatively long time and is not always suitable for all materials. The combination of PMR and generative models is believed to be able to produce PLSV learning trajectory designs that are effective in reducing learning barriers and meaningfully increasing students' understanding of concepts.

3. Proposed Method

This study uses *a design research* method that is carried out in natural conditions with the researcher acting as a *human instrument*. The subject of the study is a grade VII student of SMP Negeri 2 Mataram in the odd semester of the 2023/2024 school year. The selection of subjects was carried out through *purposive sampling techniques* based on certain criteria that are relevant to the research objectives (Creswell, 2012). The research was carried out at SMP Negeri 2 Mataram in the period of September 9–October 14, 2024.

The research data source consists of primary and secondary data. Primary data was obtained from the results of tests, observations, and direct interviews with students and teachers. Meanwhile, secondary data in the form of documents, archives, and records related to learning one-variable linear equations (Cresswel, 2012). Data collection was carried out through description tests, observation of passive participation during *the pilot experiment* and *teaching experiment*, in-depth interviews, and documentation in the form of photos, videos, and field notes. The instruments used included test questions, observation sheets, interview guidelines, field notes, and audio-visual recording devices.

The validity of the data is guaranteed through *the triangulation technique of* sources, techniques, and time. The validity of the instrument's content was tested by experts in the field of mathematics education, while reliability was obtained through cross-examination of research recordings, photographs, and records (Cohen et al., 2018). Data analysis was carried out retrospectively by comparing *the Hypothetical Learning Trajectory* (HLT) that had been previously designed with the learning process of students at the *pilot experiment* and *teaching experiment* stages. This analysis was carried out through narrative descriptions, transcription of interview results, and data grouping to produce *a Local Learning Trajectory* (LLT) that is considered effective (Prahmana, 2017; Widjaja in Prahmana, 2017).

4. Results and Discussion

4.1. Description of Research Results

This research began with a preliminary research stage to identify the *learning obstacles* faced by students in understanding the material of One Variable Linear Equation (PLSV). Based on interviews with grade VII mathematics teachers of SMP Negeri 2 Mataram, it was found that students were able to solve problems whose solutions were directly using formulas, but it was difficult to apply the PLSV concept to real problems, especially if it was associated with the previous material. This difficulty is caused by a weak understanding of concepts, limitations in visualizing, analyzing, and abstracting problems, and learning approaches that are still teacher-centered without utilizing contextual phenomena to build students' knowledge independently. In addition, contextual learning is usually delivered after students have learned formulas, rather than as an initial part of the learning process.

Based on the initial findings, the researcher designed *the PLSV Hypothetical Learning Trajectory* (HLT) in accordance with the provisions *of Permendikbudristek* No. 7 of 2022, which divides the alleged student learning trajectory into eight learning outcomes in four activities based on a realistic mathematical approach. This HLT was then tested in cycle I (*pilot experiment*) involving seven students with various abilities. The implementation of the first cycle began with *a pretest* to map students' understanding and learning barriers, followed by the implementation of HLT, and ended with a *posttest* to assess the effectiveness of the learning design. The results showed that students still had difficulty converting story problems into mathematical models, describing the results found, and presenting formal representations of PLSV appropriately.

Based on the results of the retrospective analysis of cycle I, a revision of the HLT was carried out by adding a guide to the activity of working with colored boxes and the Eiffel tower to help students connect concrete objects with mathematical models. This improvement is intended to make it easier for students to form the concept of variables and linear equations of one variable.

Cycle II (*teaching experiment*) involved a class of 44 students divided into nine heterogeneous groups. This process begins with a *pretest* to measure initial ability, continues with HLT-based learning, and closes with *a posttest*. The results of the retrospective analysis showed that HLT improvement helped reduce learning barriers, although some students still made mistakes due to lack of thorough reading of the questions. Overall, the implementation of the revised HLT is considered effective because it directs students through an appropriate learning trajectory without a leap in understanding, and supports the formation of *an appropriate Local Learning Trajectory* (LLT) for PLSV learning in grade VII (Prahmana, 2017).

4.2. Discussion and Findings

Based on the results of the study, learning obstacles in learning One-Variable Linear Equations (PLSV) in grade VII are mainly included in the category of epistemological obstacles (Brousseau, 2002), which is the limitation of students in applying the concepts that have been learned to various contexts. These obstacles can be seen from the difficulty of students converting the problem into a mathematical model form, generalizing the results, representing the form of the equation, and understanding the meaning of the information given. Some students also experience obstacles in applying addition, subtraction, multiplication, or division operations in the context of linear equations, resulting in a low understanding of concepts (Wijaya et al., 2019).

To overcome these obstacles, this study developed a learning trajectory based on a realistic mathematical approach to generative models. Learning trajectory is defined as a set of learning objectives, instructional tasks, and student thought flow that are structured to aid the development of comprehension (Clements & Sarama, 2020; Surya, 2018). The designed trajectory contains four main activities: introduction of concepts through concrete context (abstraction), formulation of concepts, validation through discussion and exercise, and application of concepts to contextual problems. This series facilitates students to build knowledge gradually, so that their understanding of PLSV can improve significantly without experiencing a hindering concept jump (Gravemeijer, 2004).

The effectiveness of the learning trajectory design can be seen from the results of retrospective analysis in two learning cycles. In the process, there was a strengthening of abstraction skills in the initial activities, mastery of formulation and validation in the intermediate stage, and the strengthening of the application of concepts in the final stage. As a result, there was a significant decrease in the number of learning barriers experienced by students, as shown by the comparison of pretest–posttest scores in both cycles, as well as improvements in students' answers in representation and solving contextual problems. Post-learning interviews also revealed that students responded positively to this design because it helped them understand PLSV in a more concrete and meaningful way.

5. Conclusions

Based on the results of the study, the learning obstacles experienced by grade VII students in learning one-variable linear equations are classified as *epistemological obstacles*, namely the difficulty of applying mathematical concepts that have been learned to various situations or problems. Obstacles identified include difficulties in using addition, subtraction, multiplication, and division operations to recognize the concept of a single-variable linear equation; limitations in identifying and explaining concepts on real objects; difficulty in making representations of mathematical models; obstacles in generalizing the form of equations; and limitations in understanding the meaning of information in the results of representation. To overcome this, this study developed *a Local Learning Trajectory* (LLT) through four learning activities: (1) observing scales to recognize concepts and solve one-variable linear equations using basic calculation operations; (2) work with colored boxes to find variable concepts and turn them into mathematical models; (3) observing objects such as the Eiffel tower or batik cloth to explain the concept of one-variable linear equations and presenting them in the form of equations; and (4) solve contextual problems in the form of flat shapes to express situations into one-variable linear equations. The resulting learning trajectory design emphasizes three main stages, namely abstraction (activities 1 and 2), concept formulation (activity 3), and concept application (activity 4). The application of this design provides students with the opportunity to think, discover, and understand the concept of linear equations independently through mathematical activities that involve real context. Thus, students no longer find this material difficult, as concepts are acquired through hands-on experience, which ultimately has a positive impact on reducing learning barriers to one-variable linear equation material in grade VII.

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